Making Möbius Strips

Imogen I. Morris

CISA Lunch Talk

8th November 2017

Activity 1: A Parodoxical Surface

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2. If you colour in a band of paper that has no twist you should be able to use two colours without touching, one for each side.

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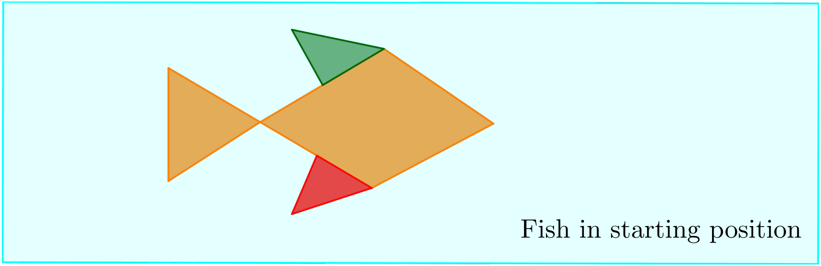
- This is equivalent to whether the surface contains a subset that is homeomorphic to the Möbius strip.

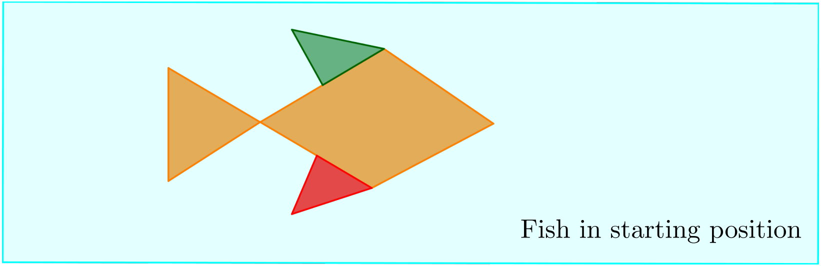
Nonorientability of the Möbius strip

Imagine that the Möbius strip is a thin band of water with a fish swimming along it.

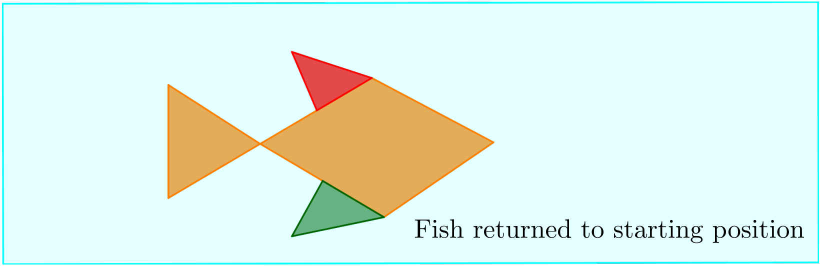


Nonorientability of the Möbius strip

Once the fish swims back to where it began, we find that the fish has been reflected.

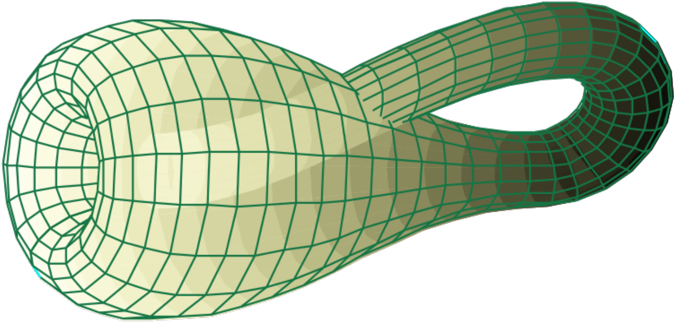


Fish in starting position



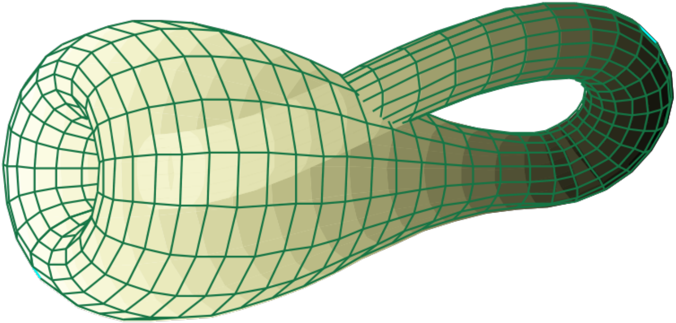
Fish returned to starting position

Klein Bottle



Picture from Wikipedia

Klein Bottle



Picture from Wikipedia

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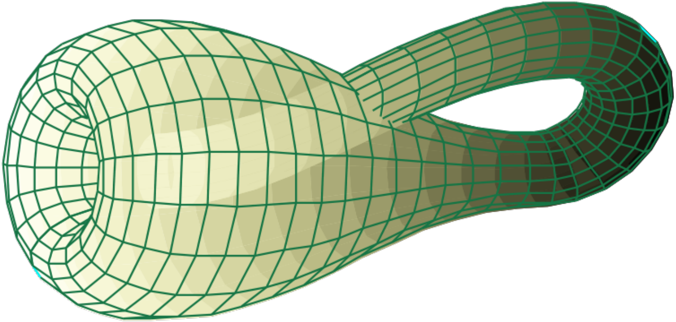
Klein Bottle



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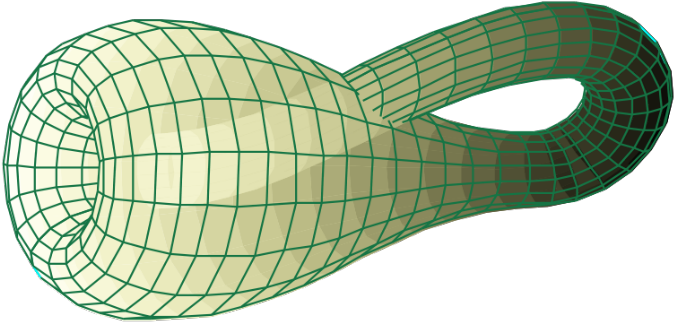
- When we embed the Klein bottle in three dimensions it must have a self-intersection.

Klein Bottle



Picture from Wikipedia

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- When we embed the Klein bottle in three dimensions it must have a self-intersection.
- Kleinsche Fläche (“Klein surface”) and then misinterpreted as Kleinsche Flasche.

Klein Bottle (cont.)

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- In three dimensional Euclidean space we can define the normal to a surface using the right-hand rule.
- This allows us to distinguish between the inward pointing and outward pointing normal.
- This only makes sense when we have a consistent choice of clockwise, i.e. if the surface is orientable.
- As a Klein bottle is nonorientable, we cannot distinguish between its inside and outside.

Activity 2: Infinite Descent

1. Make a nice, fat, Möbius strip. Try cutting it exactly down the middle, following along the band, rather than across. What do you get? Can you understand why this happens? Now try cutting the result exactly down the middle. Repeat until it becomes too thin to continue.

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2. Make another fat Möbius strip. Try cutting it in thirds, again by following along the band. You might need to mark the thirds on the band first unless you can easily judge them whilst cutting. What do you get? Can you understand why this happens?

Activity 3: Hearts and Squares

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1. Make two simple bands of paper. Now tape one perpendicularly to the other and cut them exactly down the centre, as in Activity 1. What do you get?
2. Make two Möbius strips, but make the twists in opposite directions. Try to work out if they are different or the same. Again tape one perpendicularly to the other and cut them exactly down the centre. What do you get? Try making two Möbius strips with the twists in the same directions and repeat.

Further Exploration

Knit an intrinsic-twist Möbius strip or Klein bottle hat: patterns available on the website of mathematical-knitter sarah-marie belcastro
`http://www.toroidalsnark.net/`

A valentine from Möbius: video by Radcliffe Institute fellow and mathematician Tadashi Tokieda
`http://www.youtube.com/watch?v=5xLFf_SwaK4`

Cutting a Klein bottle in half: video by Numberphile, a mathematics-themed YouTube channel produced by Nottingham University
`http://www.youtube.com/watch?v=I3ZlhxaT_Ko`